Calculus Information Summary

Trigonometry:

(1) Identities

$\sin 2x = 2\sin x\cos x$

$$\cos 2x = \begin{cases} \cos^2 x - \sin^2 x \\ 2\cos^2 x - 1 \\ 1 - 2\sin^2 x \end{cases}$$

$\sin^2 x + \cos^2 x = 1$

$\sec^2 x = \tan^2 x + 1$ or $\tan^2 x = \sec^2 x - 1$

$\csc^2 x = \cot^2 x + 1$ or $\cot^2 x = \csc^2 x - 1$

(2) Derivatives:

Function	*Derivative*
$y = \sin x$	$y' = \cos x$
$y = \cos x$	$y' = -\sin x$
$y = \tan x$	$y' = \sec^2 x$
$y = \sec x$	$y' = \sec x \tan x$
$y = \csc x$	$y' = -\csc x \cot x$
$y = \cot x$	$y' = -\csc^2 x$
$y = \arcsin x$	$y' = \frac{1}{\sqrt{1-x^2}}$
$y = \arctan x$	$y' = \frac{1}{1+x^2}$
$y = arc\sec x$	$y' = \frac{1}{\lvert x\rvert\sqrt{x^2-1}}$

(3) Trig Integrals:

$\int \sin x dx = -\cos x + C$

$\int \cos x dx = \sin x + C$

$\int \tan x dx = \ln\lvert\sec x\rvert + C = -\ln\lvert\cos x\rvert + C$

$\int \sec x dx = \ln\lvert\sec x + \tan x\rvert + C$

$\int \csc x dx = -\ln\lvert\csc x + \cot x\rvert + C$

$\int \cot x dx = \ln\lvert\sin x\rvert + C$

$\int \sin^2 x dx = \frac{1}{2}x - \frac{1}{4}\sin 2x + C$ because $\sin^2 x = \frac{1-\cos(2x)}{2}$

$\int \cos^2 x dx = \frac{1}{2}x + \frac{1}{4}\sin 2x + C$ because $\cos^2 x = \frac{1+\cos(2x)}{2}$

$\int \sec^2 x dx = \tan x + C$

$\int \csc^2 x dx = -\cot x + C$

$\int \tan^2 x dx = \int\left(\sec^2 x - 1\right)dx = \tan x - x + C$

$\int \cot^2 x dx = \int\left(\csc^2 x - 1\right)dx = -\cot x - x + C$

(4) **More Integrals:**

$\int \frac{du}{u} = \ln|u| + C$

$\int e^u du = e^u + C$

$\int a^u du = \frac{a^u}{\ln|a|} + C$

$\int \frac{du}{\sqrt{a^2 - u^2}} = \arcsin \frac{u}{a} + C$

$\int \frac{du}{a^2 + u^2} = \frac{1}{a} \arctan \frac{u}{a} + C$

$\int \frac{du}{u\sqrt{u^2 - a^2}} = \frac{1}{a} arc\sec \frac{|u|}{a} + C$

Differentiation Rules

(1) **Product Rule:**

$h(x) = f(x)g(x)$ $\qquad$ $h'(x) = f'(x)g(x) + f(x)g'(x)$

(2) **Quotient Rule:**

$h(x) = \frac{f(x)}{g(x)}$ $\qquad$ $h'(x) = \frac{g(x)f'(x) - f(x)g'(x)}{(g(x))^2}$

(3) **Chain Rule**

$h(x) = f(g(x))$ $\qquad$ $h'(x) = f'(g(x))g'(x)$

Justification for

(1) **Relative Extrema:**

1st Derivative Test

2nd Derivative Test

(2) **Point of Inflection**

Curve is smooth and $f''(x)$ changes sign at the point.

Average rate of change over an interval [a, b] is slope of the secant line.

$$\frac{f(b) - f(a)}{b - a}$$

Average value of a function over an interval [a, b]:

$$\overline{f(x)} = \frac{1}{b-a} \int_a^b f(x)\,dx$$

Limits

(1) **L'Hopital's Rule**

$\lim \frac{f(x)}{g(x)} = \frac{\infty}{\infty} \text{ or } \frac{0}{0} = \lim \frac{f'(x)}{g'(x)}$

(2) **Derivative:**

$$\lim_{h\to 0} \frac{f(x+h)-f(x)}{h} \Leftrightarrow f'(x)$$

$$\lim_{x\to a} \frac{f(x)-f(a)}{x-a} \Leftrightarrow f'(a)$$

Continuity Theorems

(1) **MVT (Mean Value Theorem)** If $f(x)$ is differentiable over the interval, [a, b], then there exists a point c on the interval such that the derivative at c is equal to the slope of the secant line.

$$f'(c) = \frac{f(b)-f(a)}{b-a}$$