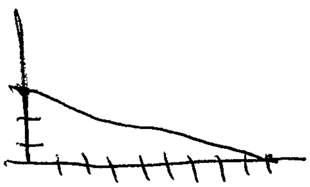
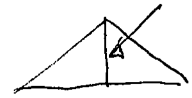
Review Problems Name Key

1) The base of a solid region in the first quadrant bounded by the x-axis, the y-axis, and the line x + 3y = 9. If cross sections of the solid perpendicular to the x-axis are isosceles right triangles, what is the volume of the solid?



$y = -\frac{1}{3}x + 3$



$\frac{1}{4}\int_0^9 (-\frac{1}{3}x+3)^2\,dx$

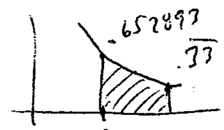
$\frac{1}{4}\int_0^9 (\frac{1}{9}x^2 - 2x + 9)\,dx$

$\frac{1}{4}\left(\frac{1}{27}x^3 - x^2 + 9x\right]_0^9$

$\frac{1}{4}(27 - 81 + 81) = \frac{27}{4} = 6.75$

For problems 2 & 3, let R be the region in the first quadrant enclosed by $y = \frac{1}{x-1}$, x = 3, and x = 4.

2) What is the area of R?



$\int_3^4 \frac{1}{x-1}\,dx$ $u = x-1$ $du = dx$

$\int_2^3 \frac{1}{u}\,du$ $\ln u\Big]_2^3$ $\ln\frac{3}{2}$

3) Find the volume of the solid generated when R is revolved about the line y = 1.

$\pi\int_3^4 1^2 - \left(\frac{x-2}{x-1}\right)^2 dx$ $u = x-1$ $x = u+1$ $du = dx$

$R = 1$

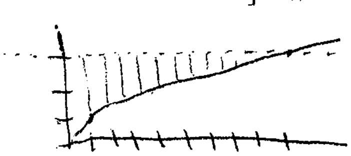
$r = 1 - \frac{1}{x-1} = \frac{x-1-1}{x-1} = \frac{x-2}{x-1}$

$\pi\int_2^3 1 - \left(\frac{u-1}{u}\right)^2 du = \int_2^3 1 - (1-\frac{1}{u})^2\,du$

$\int_2^3 \frac{2}{u} - \frac{1}{u^2}\,du$ $\left(2\ln\frac{3}{2} + \frac{1}{3} - \frac{1}{2}\right)\pi = \left(2\ln\frac{3}{2} + -\frac{1}{6}\right)\pi$

4) The region enclosed by the x-axis, the line y = 3, and the curve $y = \sqrt{x}$ is rotated about the x-axis. What is the volume of the solid generated? Then about the y-axis.

A] 3π B] $2\sqrt{3}\pi$ C] $\frac{9}{2}\pi$ D] 9π E] $\frac{81}{2}\pi$



R = 3
r = $\sqrt{x}$
washers

$\pi\int_0^9 (3^2 - \sqrt{x}^2)\,dx$

$\pi\left(9x - \frac{x^2}{2}\right]_0^9 \Rightarrow \pi(81 - \frac{81}{2}) = \frac{81}{2}\pi$

$x = y^2$
disks

$\pi\int_0^3 y^2\,dy$

$\pi\left(\frac{1}{3}y^3\right)\Big]_0^3 \rightarrow 9\pi$

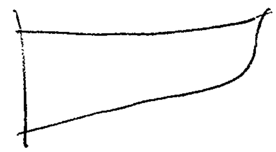
SET 26

1. 36π
2. 2π
3. $\frac{16\pi}{15}$
4. 2π
5. $\frac{16\pi}{5}$
11. $\frac{256}{3}$
12. $\frac{128}{15}$

5) Let R be the region in the first quadrant enclosed by the graph of $y=(x+1)^{\frac{1}{3}}$, the line $y=7$, the x-axis, and the y-axis. Set up the problem to solve the volume of the solid generated when R is revolved about the y-axis.

$\pi\int_1^7 (y^3-1)^2\,dy$

$\frac{815184}{7}\pi$



6) The average value of $\cos x$ on the interval [-3, 5] is

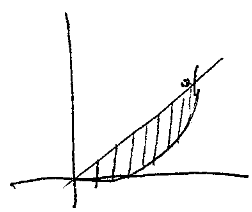
A] $\frac{\sin 5-\sin 3}{8}$ B] $\frac{\sin 5-\sin 3}{2}$ C] $\frac{\sin 3-\sin 5}{2}$ D] $\frac{\sin 3+\sin 5}{2}$ E] $\frac{\sin 3+\sin 5}{8}$

$\frac{1}{8}\int_{-3}^{5}\cos x\,dx$ $\quad \frac{1}{8}\sin x\Big]_{-3}^{5}$ $\quad \frac{\sin 5-\sin(-3)}{8}$

$\frac{\sin 5+\sin(3)}{8}$

7) The area of the region enclosed by the graph of $y=x^2$ and $y=x$ is

A] $\frac{1}{6}$ B] $\frac{1}{3}$ C] $\frac{1}{2}$ D] $\frac{5}{6}$ E] 1



$\int_0^1 (x-x^2)\,dx$

$\frac{1}{2}x^2-\frac{1}{3}x^3\Big]_0^1$ $\quad \frac{1}{2}-\frac{1}{3}=\frac{1}{6}$

8) Find the volume of the solid whose base is the region defined in problem 7 and whose cross sections cut the plane perpendicular to the x-axis and are semicircles.

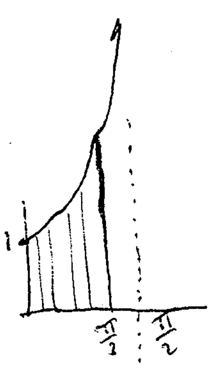
$A=\frac{1}{2}\pi r^2 \quad r=\frac{1}{2}(x-x^2)$ $\quad \frac{1}{3}-\frac{1}{2}+\frac{1}{5}$

$10-15+6$

$\frac{1}{2}\pi\int_0^1\left(\frac{x-x^2}{2}\right)^2dx$ $\quad \frac{1}{8}\pi\int_0^1(x^2-2x^3+x^4)\,dx$ $\quad \frac{1}{8}\pi\left(\frac{x^3}{3}-\frac{x^4}{2}+\frac{x^5}{5}\right)\Big]_0^1$

$\frac{\pi}{240}$

$\left(\frac{1}{2}\ \frac{\sqrt{2}}{2}\right)$



9) What is the volume of the solid generated by rotating about the x-axis the region enclosed by the curve $y=\sec x$ and the lines $x=0$, $y=0$, and $x=\frac{\pi}{3}$?

A] $\frac{\pi}{\sqrt{3}}$ B] π C] $\pi\sqrt{3}$ D] $\frac{8\pi}{3}$ E] $\pi\ln\left(\frac{1}{2}+\sqrt{3}\right)$

$\pi\int_0^{\frac{\pi}{3}}(\sec x)^2dx \rightarrow \pi\int_0^{\frac{\pi}{3}}\sec^2 x\,dx \rightarrow \pi(\tan x\Big]_0^{\frac{\pi}{3}} \rightarrow \pi(\sqrt{3}-0) \Rightarrow \pi\sqrt{3}$